NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science ; Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BAMS	**LEVEL:** 6
COURSE CODE: ODE602S	**COURSE NAME:** ORDINARY DIFFERENTIAL EQUATIONS
SESSION: NOVEMBER 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr A. S EEGUNJOBI
MODERATOR:	Dr I.K.O AJIBOLA

INSTRUCTIONS

1. Answer ANY FOUR (4) questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

QUESTION 1 [25marks]

1. (a) Find the general solution of the following differential equations:

i.
$$y'(x)\sin x + y(x)\cos x = 2e^{x}, \quad y(1) = a, \quad 0 < x < \pi$$
(5)

ii.
$$\frac{dy}{dx} = \frac{3x^2+1}{3y^2-12y}, \quad y(0) = 1$$
and determine the interval in which the solution is valid. (5)

iii.
$$\frac{dy}{dx} = \frac{ay(x)+b}{cy(x)+d}$$
where $a; b; c; d$ are constants. (5)

(b) i. A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After 5 hours the population has increased to 400. When will the population reach 4,000? (5)

ii. Solve the differential equation
$$xdy - (x^2+3y)dx = 0.$$
(5)

QUESTION 2 [25 marks]

2. (a) i. If $y_1(x)$ and $y_2(x)$ are two solutions of second order homogeneous differential equation of the form
$$y''(x) + p(x)y'(x) + q(x)y(x) = 0$$
where $p(x)$ and $q(x)$ are continuous on an open interval I, then show that
$$W(y_1(x), y_2(x)) = ce^{-\int p(x)dx}$$
where c is a constant. (6)

ii. Use reduction of order method to find $y_2(x)$ if
$$x^2y'' - 3xy' + 4y = 0; \quad y_1(x) = x^2$$
(6)

(b) Solve the following:

i.
$$y''(x) + 2y'(x) + 10y(x) = 0$$
(6)

ii.
$$y''(x) - 3y'(x) - 4y(x) = 0, \quad y(0) = 2, \quad y'(0) = 3$$
(7)

QUESTION 3 [25 marks]

3. (a) Solve the Euler equation

$$x^2y''(x) + 3xy'(x) + 2y(x) = 0, \quad y(1) = 1, \quad y'(1) = 0$$

(9)

(b) Solve the following differential equations by method of variation of parameters $y''(x) + y(x) = \tan x$ (8)

(c) Solve the following differential equations by method of undetermined coefficient

$$y''(x) + 2y'(x) + 2y(x) = -e^x(5x - 11), y(0) = -1, \quad y'(0) = -3$$

(8)

QUESTION 4 [25 marks]

4. (a) i. Solve using Laplace transform

$$y'(t) + 2y(t) = 4te^{-2t}, \qquad y(0) = -3$$

(6)

ii. If $\mathcal{L}\{f(t)\} = F(s)$, show that

$$\mathcal{L}\{e^{bt}f(at)\} = \frac{1}{a}F\left(\frac{s-b}{a}\right)$$

(6)

iii. Find

$$\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+2s+5}\right\}$$

(6)

(b) Solve the following differential equation by using Laplace transform

$$y''(t) + y'(t) + y(t) = \sin t, \quad y(0) = 1, \quad y'(0) = -1$$

(7)

QUESTION 5 [25 marks]

5. (a) Solve using Laplace transform

$$y'' + y = 2\cos t, \quad y(0) = 0, \quad y'(0) = 0$$

(10)

(b) i. Find the value of α that will make

$$(ye^{2xy} + x)dx + \alpha xe^{2xy}dy = 0$$

exact? (7)

ii. Hence or otherwise solve (8)

End of Exam!